Enrollment No. ____________

Shree Manibhai Virani and Smt. Navalben Virani Science College (Autonomous), Rajkot
Affiliated to Saurashtra University, Rajkot

SEMESTER END EXAMINATION APRIL - 2018

M.Sc. Mathematics

16PMTCC18 – NUMBER THEORY -II

Duration of Exam – 3 hrs | ***Semester – IV*** | ***Max. Marks – 70***

***Part A** (5X2= 10 marks)*
Answer **ALL** questions

1. Define: Primitive P-triplet.
2. If $a|b$ is farey fraction then show that $(a,b)=1$.
3. Expand $\frac{17}{3}$ in terms of simple continued fraction.
4. Convert $< 1,2,1,2 \ldots >$.
5. Draw Ferrers Graph of $n=5$.

***Part B** (5X5 = 25 marks)*
Answer **ALL** questions

6a. Show that the Diophantine equation $ax+by=c$ $(a^2+b^2\neq 0)$ has a solution if and only if d/c where $d=(a,b)$.

OR

6b. Find General Solution of $63x+7=23y$.

7a. If $\frac{\mathrm{a}}{\mathrm{b}}$ and $\frac{\mathrm{c}}{\mathrm{d}}$ are consecutive farey fraction in the $\mathrm{n^{th}}$ row then show that $|\mathrm{ad}-\mathrm{bc}|=1$.

OR

7b. If θ is an irrational number then show that there are infinitely many rational number $\frac{a}{b}$ such that $\left|\theta-\frac{a}{b}\right|<\frac{1}{b^2}$. (Farey Fraction)

8a. Show that the value of a finite simple continued fraction is a rational number.

OR

8b. Convert $< 2,3,6,1 >$ into rational number and expand $\frac{3}{17}$.

9a. Find any three positive solution of $x^2-3y^2=1$.

OR

9b. Prove that there are infinitely many positive integers n such that $1+2+3+\cdots+n$ have perfect square.

10a. Define Partition and write all partition of $n=6$.

OR

10b. Show that $\prod_{j=1}^{\infty} 1+x^j=\prod_{n=1}^{\infty}(1+x^{2n-1})^{-1}$.

<u>Part C</u> (5X7 = 35 marks)

Answer **<u>ALL</u>** questions

11a. Define Perfect square and show that if $u\ and\ v$ are relatively prime positive integer whose product uv is perfect square then show that $u\ and\ v$ are perfect square.

OR

11b. The primitive positive solution of Pythagorean triplet $x^2 + y^2 = z^2$ with y is even then show that there are positive integer r and s such that $r > s \geq 1$, one of $r\ and\ s$ have opposite parity, $(r, s) = 1, z = r^2 + s^2, y = 2rs$ and $x = r^2 - s^2$.

12a. If θ is an irrational number then there are infinitely many rational $\frac{a}{b} \ni \left|\theta - \frac{a}{b}\right| < \frac{1}{b^2}$.

OR

12b. If $\frac{a}{b}$ and $\frac{c}{d}$ are consecutive Farey fraction in the any row then show that $\frac{a+c}{b+d}$ is the unique rational number between $\frac{a}{b}$ and $\frac{c}{d}$ such that its denominator is the smallest among all rational between $\frac{a}{b}$ and $\frac{c}{d}$.

13a. If θ is an irrational number then show that there are infinitely many rational number $\frac{a}{b}$ such that $\left|\theta - \frac{a}{b}\right| < \frac{1}{\sqrt{5}b^2}$ using continued fraction.

OR

13b. Show that any rational number can be expressed as a value of some simple finite continued fraction.

14a. Show that an irrational no θ is quadratic irrational if and only if its simple continued fraction is periodic.

OR

14b. Let θ be a quadratic irrational then the continued fraction expansion of $\theta\ purily$ periodic if and only if 1) $\theta > 1$ and 2) $-1 < \theta' < 0$.

15a. Draw the Farrers Graph for $n = 8$.

OR

15b. Define Partition and find write all partition of $n = 7$.